1986 Intermag Conference - April 1986

A permanent-magnet field source for the production of circularly polarized radiation via helical free-electron lasers

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Abstract

A relativistic electron-beam laser designed at the Naval Research Laboratory produces circularly polarized 23 GHz radiation by means of a transverse magnetic field of which the orientation rotates continuously in the ϕ direction with displacement along the z axis. A field strength of about 500 Oe is presently provided by a bifilar solenoid carrying a current of 200 amperes. It is desirable to eliminate the necessity of such high currents and the attendant bulky power supplies by means of a permanent-magnet field source. This can be accomplished by a tubular magnetic structure of rectangular cross section, which is twisted progressively about its z axis with the desired pitch of field rotation (2π radians in 2.5 to 6 cm). By means of a sheathing of rare earth permanent magnets oriented normally to the magnets supplying the working flux, the magnetic field can be confined to the twisted rectangular tube through which passes the cylindrical tube carrying the electron beam. Also discussed is the design of an outer permanent-magnet structure for the supply of a solenoidal focusing field of 3 kOe.

Introduction

Recently, there has been considerable interest in the application of relativistic electron beams to high-power, broad-band radiation sources for micro- and millimeter-wave radars. It is often desirable in such devices to replace solenoidal magnetic field sources and their cumbersome attendant power supplies with more compact and tractable permanent-magnet configurations. For example, Figure 1 shows a 23 GHz helical undulator (twister) developed by the Naval Research Laboratory.[1,2] It employs a bifilar coil carrying a current of 200 amperes to produce a helically varying transverse field of 500 oersteds. A cylindrical solenoid carrying 60 amperes provides a 3,000 oersted field to focus the electron beam. It is the purpose of the present work to replace both electrical coils with permanent-magnet structures.

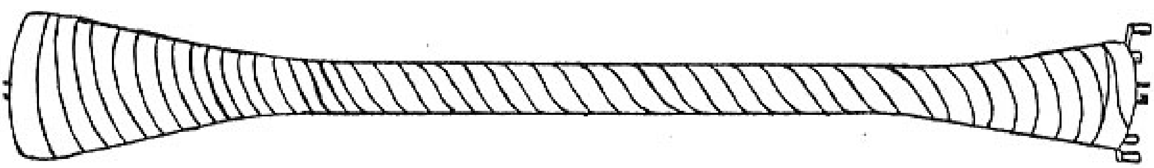

Figure 1. Bifilar helical coil flared at ends to prevent abrupt rise in field there.

Design of a cladded structure

A simple magnetic pole distribution that would produce the desired transverse field would be that of two parallel infinite pole sheets of opposite sign, twisted about the cylindrical beam space with the appropriate helical pitch. (See Figure 2.) The on-axis transverse field of such a distribution is easy to calculate and is given by

$$H_w = H_o e^{-ka} \quad (1)$$

where H_o is the field for an untwisted structure, k is 2π times the number of rotational periods per centimeter, and a is the radius of the cylindrical tube about which the planes are twisted.

To produce the same flux distribution in the vicinity of the axis with a mechanically realizable structure, we consider the arrangements shown in Figures 2c and 2d. Rare earth permanent magnets m_s supply the flux needed to produce a transverse field H_w in the infinitely long working space W. The iron pole pieces p ensure that flux is distributed uniformly across the x-z plane of W. To confine all of the flux to the working space,

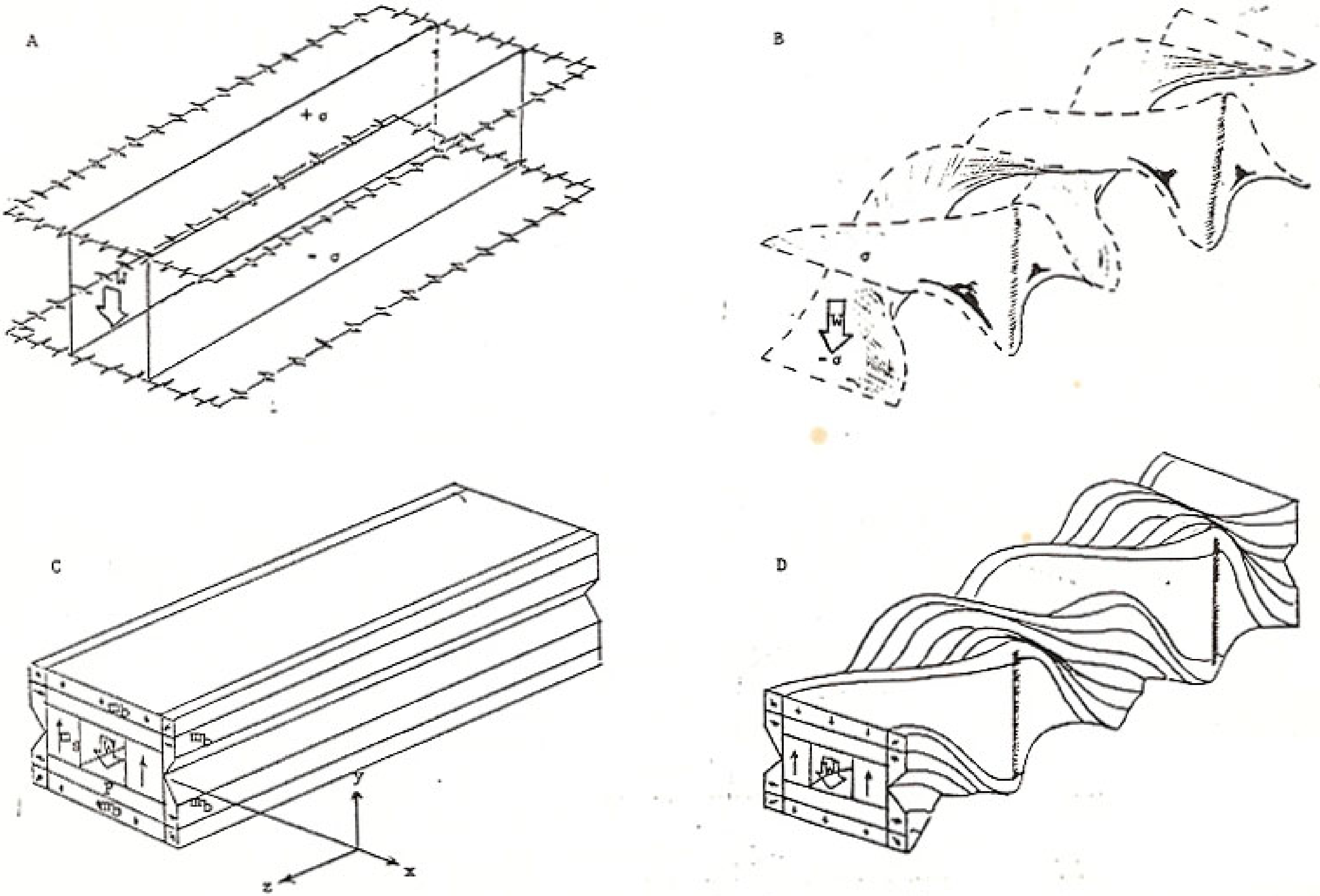

Figure 2. Production of a twister field. A. Parallel infinite planes. B. Twisted infinite planes. C. Straight cladded structure. D. Twisted cladded structure. Small arrows show direction of magnetization. Large arrows show direction of magnetic field H_w.

peripheral bucking magnets m_b are oriented with their magnetizations normal to magnets m_s and pole pieces P in such a way that all points on the outer surfaces of the resulting structure are at the same magnetic potential. If $y = 0$ is chosen as the plane of zero potential, then the magnetic potential F increases linearly in the y direction as

$$F_{ms}(y) = H_w y = H_{ms} y \tag{2}$$

because by applying Maxwell's equation to the inner boundaries of magnets m_s we see that

$$H_w = H_{ms}. \tag{3}$$

Since the bucking magnets are to carry no flux, their internal fields must be equal to their coercivities. This means that the potential F_b between the poles of the bucking magnets is

$$F_b(y) = -H_c t_b(y) \tag{4}$$

where t_b is the distance between poles. If we are to have zero potential everywhere on the structure's outer surface, F_b must exactly cancel F_m, which means that, according to (2) and (4),

$$t_b(y) = H_w y/H_c, \tag{5}$$

so the bucking magnets cladding m_s have triangular cross sections as shown. Elsewhere, the bucking magnets have the maximum thickness given by (5), namely,

$$t_b\ (L) = H_w L/H_c \tag{6}$$

where L is half the y dimension of W. The only remaining dimension to be determined is the combined thickness t_x in the x direction of the supply magnets m_s. For the system to be self-consistent, the flux/unit length ϕ_{ms} supplied by m_s must equal the flux in W

$$\phi_{ms} = \phi_w = H_w W_x \tag{7}$$

where W_x is the x dimension of the working space W, but

$$\phi_{ms} = B_{ms}\ t_x \tag{8}$$

where B_{ms} is the flux density in m_s and we have

$$t_x = H_w\ W_w/B_{ms}. \tag{9}$$

For a magnetic material with a square hysteresis loop, B_{ms} is given by

$$B_{ms} = H_{ms} + B_r \tag{10}$$

where B_r is the remanence of the magnet material and $H_{ms} = H_w$ by equation (3). Substituting (3) and (10) in (9) yields

$$t_x = \frac{H_w\ W_x}{H_w + B_r} = \frac{W_x}{1 + B_r/H_w} \tag{11}$$

and all of the dimensions of the structure are determined in terms of specified dimensions, fields, and material properties. Figure 3 shows a sectional flux plot of an untwisted structure, designed to give a uniform field of 1.5 kOe.

To design a structure of the type shown in Figure 2c, which produces the same field over a region W as the infinite parallel pole sheets of density σ in Figure 2a, the value

$$H_w = 4\pi\sigma \tag{12}$$

is substituted in equations (5), (6), and (9).

Figure 4 shows how the transverse field declines with increasing ratio of period-to-tube radius. To restore to the twisted structure the transverse field that it had when it was untwisted, the original pole density σ must be increased by a factor of e^{ka}. An equivalent field restoration of the finite structure of Figure 2d can be made by increasing the supply magnet thickness t_x by the same factor. Because the transverse field H_w will be the same as in the untwisted structure, the thickness t_b of the cladding magnets remains the same. It is clear from Figure 4 that, if high twist rates are desired for high-frequency applications, the working space radius will have to be lessened if the necessary magnetic field strength is to be achieved with available materials. The twist also gives rise to an axially

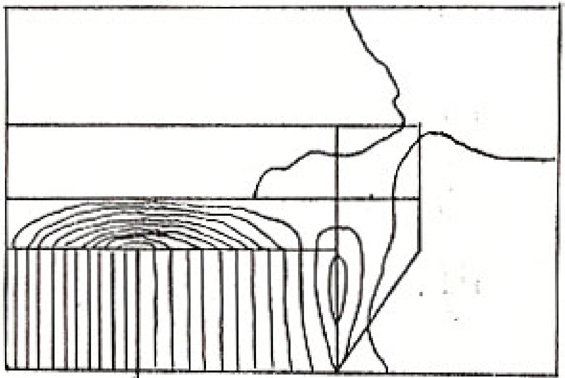

Figure 3. Quarter section flux plot of Figure 2c.

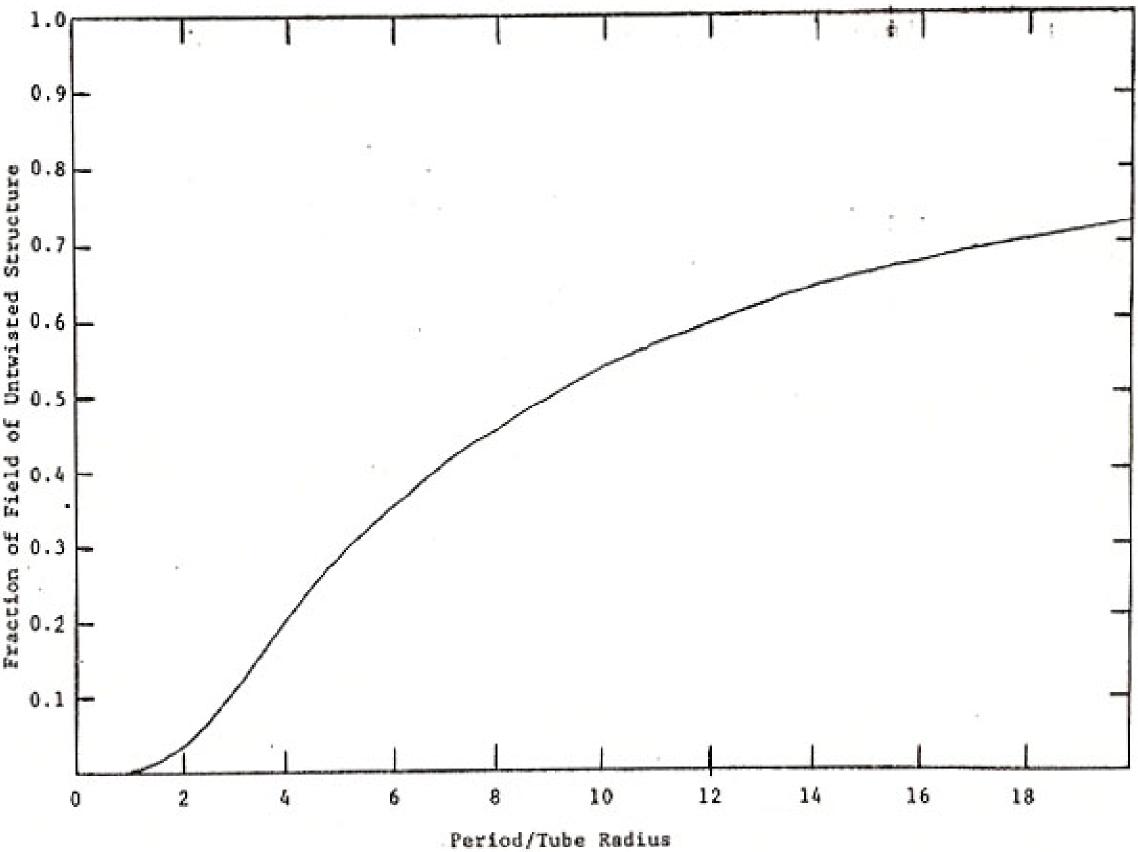

Figure 4. N dependence of transverse field of twisted cladded structure.

oriented field component that is zero on the center axis and increases with radial distance from the axis. The axial components occur in two distinct intermeshing helical regions in which the fields have opposite signs, as in Figure 5. Such axial fields also arise when a bifilar coil is used as a field source, and, while they do not prevent operation of the device, their full ramifications are not yet completely understood.

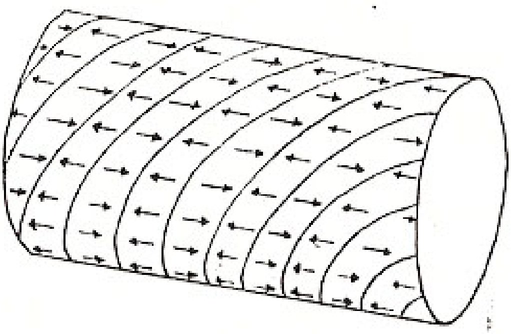

Figure 5. Axial magnetic field structure at periphery of electron-beam tube.

A dipolar structure

An alternate structure for producing a helical transverse field is pictured in Figure 6. Its helically stacked segments are of the octagonal dipolar configuration suggested by K. Halbach[3] for a variety of applications. The on-axis transverse field produced by this structure can easily be calculated by finding the equivalent surface pole distributions σ via the formula

$$\sigma = \vec{M} \cdot \hat{n} \tag{13}$$

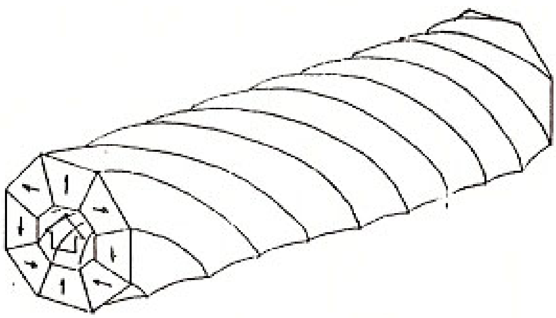

Figure 6. Twister structure with an octagonal dipolar cross section.

where $\vec{M}$ is the magnetization and $\hat{n}$ the unit vector normal to the surface at the point in question. The expressions for σ obtained from (13) can then be inserted into Coulomb's law and integrated over all surfaces to calculate the magnetic field. This procedure is possible because there are only magnetically rigid permanent-magnet materials present without the complications arising from passive magnetic materials such as iron. The fields in the cladded structure can also be found in this manner if one integrates over its equivalent distribution, shown in Figure 2b.

Comparison of the two structures

A revealing comparison of the dipolar structure with that of the cladded rectangular arrangement can be made by plotting the mass per unit length of each structure when both have the same twist period and tube radius against the desired transverse magnetic field. If this is performed for a number of pitch-to-tube radius ratios N, a family of curves, such as in Figure 7, is generated. It is apparent from these curves that, generally, the Halbach-type structure is more economical of magnetic material for lower fields and for higher values of N. If the fields at which the curves for the two structures cross are plotted against the mass per unit length, a "phase diagram" such as that of Figure 8 results. The curve divides the graph into two regions. The less bulky structure for any combination of desired field and pitch ratio is then determined by the region of the graph into which the point corresponding to that combination falls.

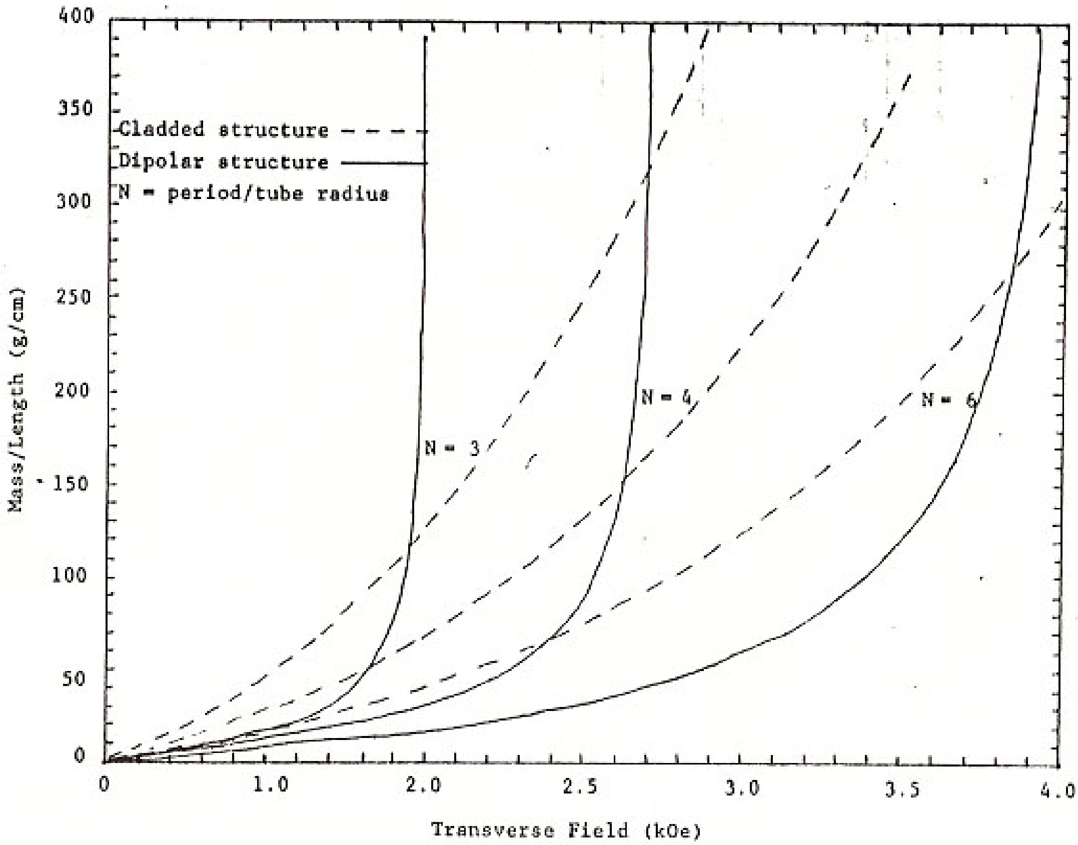

Figure 7. Comparison of masses of dipolar and cladded structures.

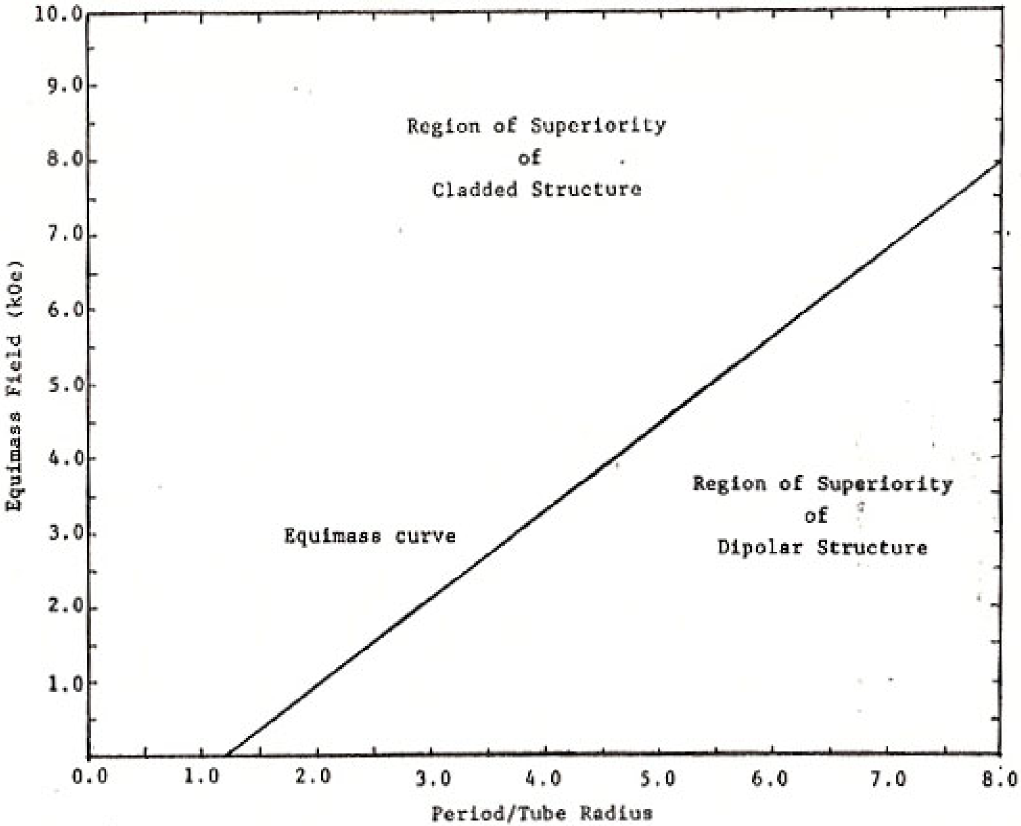

Figure 8. Diagram of regions of superiority of cladded and dipolar structures.

<u>Realizable approximations to the ideal structures</u>

No suitable known magnetic materials have the capability of withstanding the twisting necessary for the desired period-to-radius ratio N which, in the devices of interest, is of the order of two or three. The structures of Figures 2d and 6 can be approximated by a discontinuous stacking of discrete rectangular and octagonal sections, as shown in Figures 9a and 9b, respectively. Of course, the approximation can be made as good as desired by increasing the number of slices per period. However, a large number of thin sections in prohibitively expensive and impracticable. Fortunately, mechanically and economically viable devices that give a very good approximation to the ideal continuous structure can be obtained with relatively coarse sections. For example, the configuration of Figure 9b has sixteen sections per period, corresponding to the rather large rotation of 22.5 degrees between successive sections, yet, as can be seen from Figure 10, produces a transverse field that differs from the ideal by less than one percent.

Because all of its constituent magnets are geometrically identical, the dipolar structure tends to be cheaper and easier to manufacture than the cladded configurations. However, since the trend is likely to be towards shorter twist periods and higher magnetic fields, use of the more expensive cladded structure will probably be necessary for many applications.

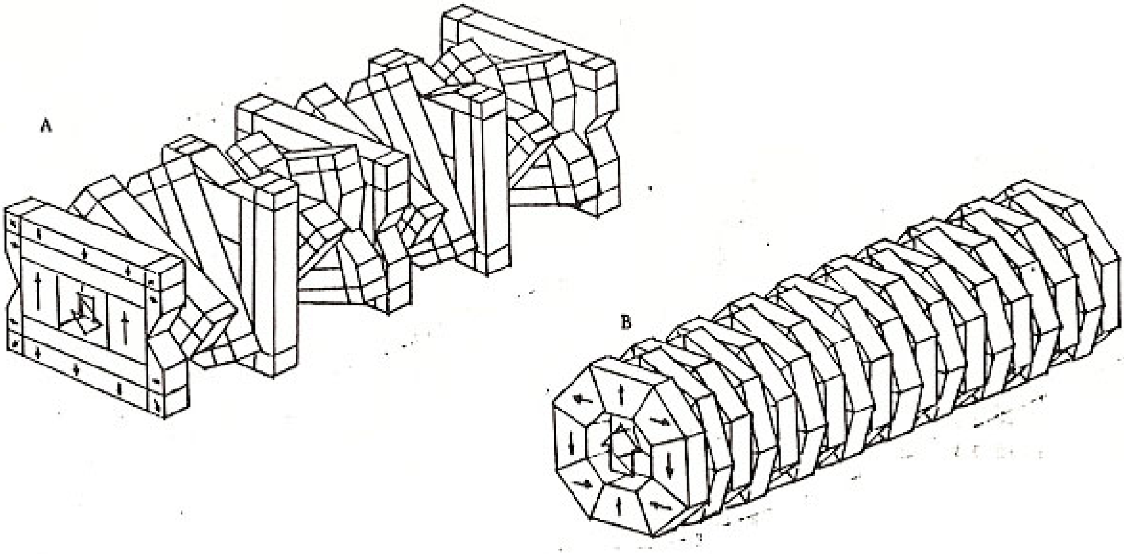

Figure 9. Realizable structures that approximate those of (a) Figure 2d and (b) Figure 6.

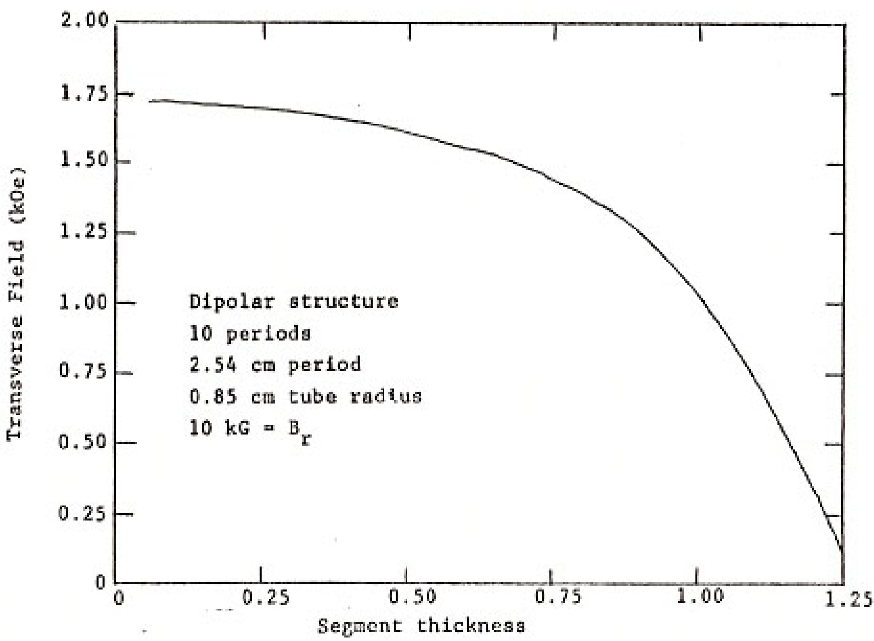

Figure 10. Effect of section thickness on transverse field in structure of Figure 9b.

Design of the focusing structure

The principles[4] invoked in the design of the focusing magnet are similar to those used for the untwisted cladded structure. An axial magnetic flux is supplied in a cylindrical space by an annular shell magnet M, and distributed via pole pieces P as shown in Figure 11. Again, radially oriented cladding m_c is used to bring the potential at every point on the outer surface to that at circumference C. Other bucking magnets m_b are placed at the ends and corners to prevent flux leakage from those points. The same set of self-consistent relationships are used to determine dimensions. The thickness of the magnet M is fixed by the condition that it provide as much flux as is needed to produce the desired field in the central cylindrical working space. To get the flux density B_m with magnet M we use equation (10), which describes the demagnetization curve for an ideal rare earth permanent magnet. The field H_m in this equation is just the field H_w desired in the working space. The maximum thickness of the cladding magnets is then given by expression (6), in which L is the half-length of W, and the structure is completely determined. The diameter of the working space is chosen so that it will accomodate the electron-beam tube and the twister structure. Holes H drilled through the pole pieces and bucking magnets at the ends, to accomodate the beam tube, should be kept as small as possible in the interest of focusing-field uniformity.

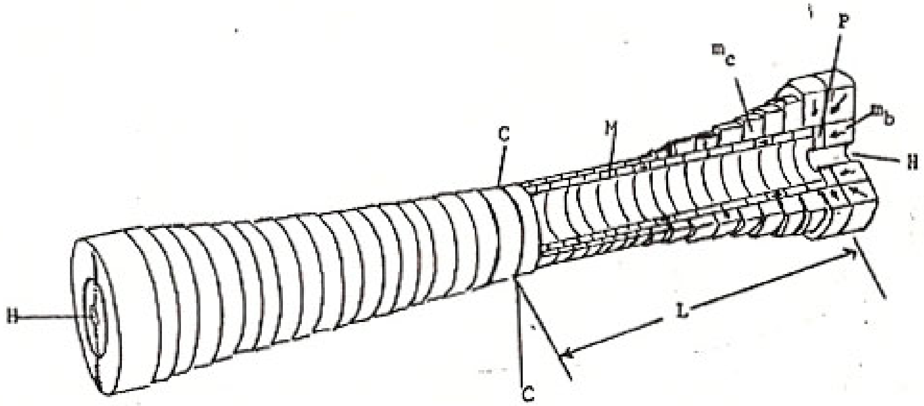

Figure 11. Permanent-magnet structure for focusing field. Small arrows show magnetization directions. Large arrow shows magnetic field direction.

Conclusions

With modern square-looped, high-energy product materials, it is possible to replace the electrical coils presently used in twisters, thus eliminating the necessity for heavy power supplies and the energy dissipation of hundreds of amperes of current. For example, the 200 ampere, 500 oersted coil presently used in an NRL prototype can be replaced by a cladded structure weighing less than five pounds. (See graphs of Figure 7 and apply them to a 100 cm long structure.) Two alternatives have been discussed, and their relative suitability for a particular application can be determined from Figure 8. Focusing solenoids can also be replaced by permanent-magnet structures.

References

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